Localization of light and second-order nonlinearity enhancement in weakly disordered one-dimensional photonic crystals

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(Received 25 November 2004; published 25 May 2005)

We show how one-dimensional photonic crystal structures which suffer from a weak random disorder in the layer lengths may give rise to strong localization of light. Using the transfer matrix method we numerically study the effects of this localization in media with a second-order nonlinearity. Localization has a deep impact on the second-harmonic generation efficiency and may give rise to very strong enhancement in correspondence to the localized wavelengths.

DOI: 10.1103/PhysRevE.71.057602 PACS number(s): 42.70.Qs, 42.25.Dd, 71.55.Jv, 42.65.Ky

Localization of light may be obtained with a mechanism very similar to the Anderson localization of electrons in solid state physics. Indeed, in the presence of a random perturbation of the material dielectric constant Anderson localization may occur if the Ioffe-Regel condition $k\ell < 1$ (where ℓ is the elastic scattering mean free path and k is the optical wave vector) is met. The basic mechanism for light localization is coherent back-scattering, that is thus considered as the precursor for Anderson localization. Although the existence of coherent back-scattering has been verified in a variety of weakly scattering systems, the mean free path ℓ is always much larger than λ, thus precluding localization. In the pioneering work by John [1] it was shown that if an underlying order in the form of some kind of periodicity is introduced, then the large scale geometric Bragg resonances modify significantly the system behavior. Indeed, localization is readily observed even with a very weak random perturbation of an otherwise perfectly periodic structure. This may be explained by noting that the k vector in the Ioffe-Regel condition must be replaced by $q=k-G$ (where G is the periodic crystal lattice vector). The periodic structure introduces Bragg reflection resonances and corresponding stop bands and, at the border of the lowest frequency band, $q \to 0$, so that the Ioffe-Regel condition is easily satisfied.

Much interest has been attracted by such a possibility, for example, for random lasers. The high optical mode density associated with light localization and the relatively short localization length opens up the possibility of obtaining efficient, extremely compact, and easy to fabricate lasers. In this Brief Report we report numerical simulations that show how it is also possible to apply the same principles to nonlinear optical wavelength conversion. We consider a one-dimensional (1D) photonic crystal waveguide that exhibits a series of Bragg resonances made from a material that possesses a second-order nonlinearity ($\chi^{(2)}$). When a certain amount of disorder is applied to the individual layer lengths that compose the crystal, we find that light localization readily occurs and gives rise to an enhancement of many orders of magnitude of the second harmonic generation (SHG) efficiency in both forward and backward directions.

We performed a series of numerical simulations using the transfer matrix technique applied to the particular case of SHG as described by Jeong and Lee [2]. This method is particularly indicated for the simulation of randomly perturbed periodic structures and has been used to describe the behavior of linear random 1D systems [3], also in the presence of gain [3,4] and second-order nonlinearity [5]. The fundamental (FF) and second-harmonic (SH) modes are approximated as plane waves with refractive indices given by the waveguide mode indices; all other modes are disregarded. The simplicity of this approach comes at the expense of a lack of information regarding the coupling to leaky or radiation modes, however, losses may be evaluated by other means, as we discuss further on. Furthermore, we performed the simulations in the undepleted pump approximation. This approach remians valid as long as the SHG conversion efficiencies remain lower than $\sim 1\%$. We therefore investigate only cases which fall in this regime without compromising the generality of our results. The particular system under investigation is an $Al_xGa_{1-x}As$ mesa waveguide described in detail elsewhere [6] and schematically represented in the inset in the lower graph of Fig. 1. The grating width varies from 500 to 700 nm between adjacent layers and the effective waveguide mode index was calculated using a commercial mode-solver [7]: $n_{\omega}^{(a),(b)}=2.89/3.06$ and $n_{2\omega}^{(a),(b)}=3.39/3.42$ are the FF and SH effective indices in each layer, respectively. The $\chi^{(2)}$ nonlinearity is taken as 100 pm/V. We started by considering a periodic structure that is made up of an elementary cell composed as (b-3a-b-3a-3b-3a) with $a=\lambda_0/4n_{\omega}^{(a)}$ and $b=\lambda_0/4n_{\omega}^{(b)}$ (λ_0 is taken as 1550 nm). Note that the arrangement of this structure does not give rise to phase-matching of any kind. We initially consider the case in which the "3b" layer (that may be viewed as a defect in an otherwise periodic structure with b-3a elementary cell) is subject to a random fluctuation, δL, that has a Gaussian distribution centered in 0 and with a standard deviation σ. The FF pump power is taken as 100 mW and the mode area in the mesa waveguide is 2 μm^2.

We point out that although the ensemble average over many realizations is usually considered in the analysis of random systems, here we are interested in the fine features of

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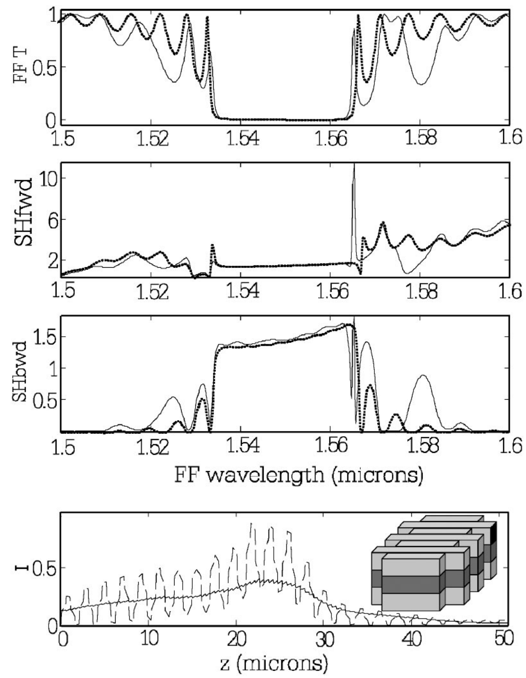

FIG. 1. Top three graphs show the transmission of the FF wave (FF T) and the generated SH in the forward (SHfwd) and backward (SHbwd) directions versus the FF wavelength for the PBG structure described in the text. The $3b$ "defect" has random length fluctuation with $\sigma=10$ nm. The bottom graph shows the FF and SHfwd ($\times 10^6$) intensity profile vs z inside the crystal for a total number of 200 layers and at 1565 nm FF wavelength. In the inset of the lower graph we show a schematic of the mesa waveguide. All vertical scales are in arbitrary units.

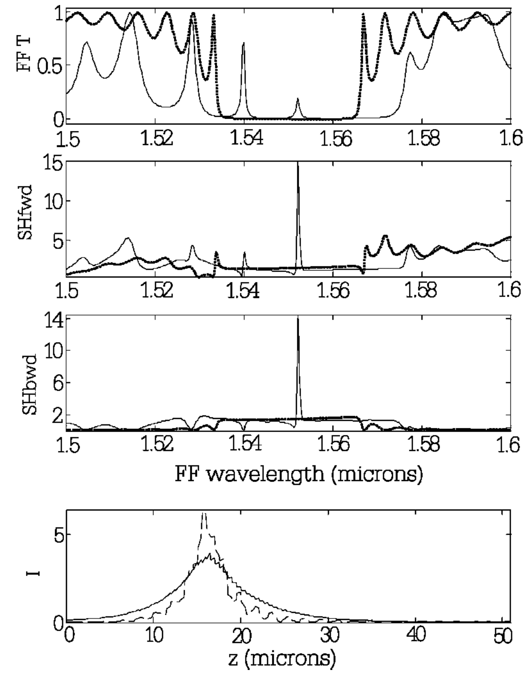

FIG. 2. Same graphs as in Fig. 1 but with $\sigma=30$ nm. The lower graph is plotted for 1552.5 nm FF wavelength. All vertical scales are in arbitrary units.

individual structures. In Fig. 1 we show in the top three graphs the fundamental transmission (FF T) and the SH generated in the forward (SHfwd) and backward (SHbwd) directions with $\sigma=10$ nm. The dotted lines in these graphs represent the response of the perfectly periodic photonic band-gap (PBG) structure, i.e., with no randomness. $\sigma=10$ nm is a relatively small, 2.6%, fluctuation of the defect layer thickness but nevertheless gives rise to some localization, with intensity peaks located in particular near the band-gap edges. In the bottom graph we show the field envelopes inside the medium for the FF wavelength (1565 nm) and for the SH. A weak localization of the FF can be seen. This in turn implies a slower energy transport velocity [8], i.e., a higher FF wavelength mode density that thus explains the corresponding enhancement of the SHG efficiency. Note that the large oscillations in the SH field amplitude are due to the phase-mismatch between FF and SH. If we increase the amplitude of the fluctuations we observe that the localization peaks tend to shift towards the center of the PBG. This feature has been observed elsewhere [3] and an example for $\sigma=30$ nm (7.9% mean fluctuation of the defect length) is shown in Fig. 2. The solid lines are for the structure with randomness and the dotted lines are without randomness. Note the well-pronounced peaks in the SHG efficiency in both forward and backward directions. This indicates the formation of a cavitylike resonance. In the lower graph, plotted for the localization wavelength, 1552.5 nm, we can see how the the FF field (solid line) is much more localized with a localization length that is smaller than the sample length. The higher mode density at this wavelength gives rise to efficient SHG (dashed line, $\times 10^6$) with more than two orders of magnitude enhancement with respect to the nonphase-matched periodic grating.

A comment on the effect of sample length. We should consider some typical lengths: the grating length L_g, the absorption length L_α, defined as the length over which the FF field is attenuated by a factor $1/e$ due to absorption or inelastic scattering, and the localization length ξ, that may be defined as the distance over which the spatial localized-field peak value decreases by $1/e$ [3]. If L_α is much larger than ξ or L_g then it will have little or no effect on the localization mechanism [9,10]. We have carried out preliminary measurements on test mesa waveguides with the same structure described here and found losses of the order of 3 to 4 dB/mm, i.e., $L_\alpha > 1$ mm (losses of the same order of magnitude have also been reported for similar structures [11]). Here $L_g \sim 50$ μm so that absorption losses may be safely neglected. Therefore, ideally, the only limiting factor on the cavity Q, or on the FF mode density, is leakage of light outside of the grating due to its small length. As the grating length is increased we observe a pronounced increase in the SHG efficiency. For example, for a structure 200 μm (800 layers) long we obtain -5 dB or higher conversion efficiencies, i.e., 300%/W (which is extremely high if compared to a typical conversion efficiency value of 500%/W in a 4 cm long periodically poled lithium niobate (PPLN) waveguide [12]) and pump depletion must be accounted for. On the other hand, if less than 200 layers are considered, ξ becomes comparable to L_g and the localization quickly disappears.

We also considered the effect of increasing σ on the en-

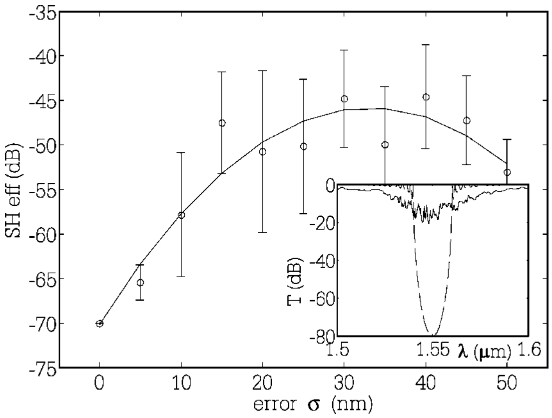

FIG. 3. Dependence of SH conversion efficiency on the error sigma for the structure described in the text. Each point is taken as the average over 20 different realizations and the error bar indicates the standard deviation around the mean value. The solid line is a polynomial fit that serves only as a guide for the eye. In the inset we plot the normalized trasmissivity versus wavelength for the perfectly periodic structure and for the structure with random fluctuations (σ=30 nm), averaged over 20 different realizations.

semble average of the SHG efficiency, when fluctuation is located every five layers. We repeated the simulations with a 300 layer structure so as to guarantee that $\xi < L_g$ and then took the average of the maximum SHG efficiency over 20 separate realizations. Figure 3 shows how an increasing σ gives an increase in SHG. However, for $\sigma > 40$ nm the efficiency starts to drop on average.

Indeed we find that for large σ randomness starts to prevail over the ordered PBG structure and the localization length starts to increase so that $\xi < L_g$ is no longer satisified on average. A very similar behavior is observed in the lasing threshold value for partially random structures with gain [3]. In the inset of Fig. 3 we show the unperturbed PBG (dashed line) that is replaced by a pseudogap for σ=30 nm.

So far we have only considered the particular case of random fluctuations on a defect layer. We note that the same effects described above were observed even if we consider a regular grating with no defects (but with the random fluctuation applied only every five layers). We can also consider the more general case in which the fluctuation affects all layers in such a way that the layers are paired and the fluctuation is taken by adding δL to one layer and subtracting δL from the adjacent one. This is very similar to what may occur during the etching process of the mesa waveguide grating where an overetch will result in thinner "fins" and wider "holes." Such a case has already been considered for σ smaller than 20 nm [5] and no localization was observed. Figure 4 shows the results for a particular realization of this type with σ =30 nm. It is interesting to note that now localization takes place albeit rather weakly and is concentrated at the bandgap edges. This situation is very similar to that observed with the randomness placed every five layers and with a much smaller σ=10 nm. This may be explained by observing that when introducing a random fluctuation on all layers we are effectively perturbing and destroying the periodic underlying PBG structure, i.e., we are destroying the balance between

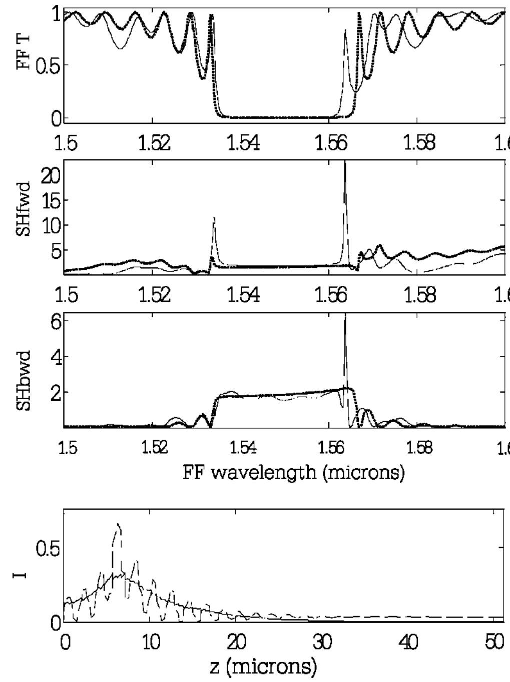

FIG. 4. Top three graphs show the transmission of the FF wave (FF T) and the generated SH in the forward (SHfwd) and backward (SHbwd) directions vs the FF wavelength for PBG structure described in the text. Each layer has a random length fluctuation (σ =30 nm) with alternating signs between successive layers. The bottom graph shows the FF and SHfwd ($\times 10^6$) intensity profile vs z, inside the crystal for a total number of 200 layers and at 1562.5 nm FF wavelength. All vertical scales are in arbitrary units.

order and disorder that gives rise to efficient localization [10]. Indeed, although in principle random 1D structures always exhibit Anderson localization (for a sufficiently long medium), the presence of an underlying periodic PBG structure greatly assists the disorder in creating wave functions that are localized over very small regions in space [9]. Any situation that is somehow intermediate between the two extreme cases illustrated here (randomness in only one layer in each elementary cell or in every layer) will show a response that is also intermediate between these, i.e., increasing the density of layers that have a random perturbation will gradually shift the structure from the "nearly ideal" PBG structure with strong and efficient localization toward that of a completely random structure with a weak underlying periodicity and a low localization enhancement.

We now consider the effect and possibility of coupling to radiation modes. The mechanism underlying the SHG enhancement presented here is very similar to that due to a mode-density increase in a cavity closed by two mirrors. It has been underlined that coupling to radiation modes at the cavity edges is the cause of high losses in such systems: one possible solution is to make the confinement gentler with a less abrupt change in passing from the cavity to the mirror [13]. The structure we have presented here may be considered as an extreme example of such an approach: we have an "effective" cavity and mirrors that are all the same object and there is no change whatsoever in passing from one to the other, i.e., a local density modes of the localization region and so we may expect to be able to neglect any additional out-of-plane losses due to localization. However, future work will consider the full three-dimensional field distribution of the optical modes in the unperturbed Bragg grating and the Fourier analysis proposed by Akahane *et al.* [13] shall be used to study and reduce coupling losses to radiation modes.

One last issue regards the maximum Q actually achievable. Very high powers may be achieved inside the material that may, in turn, lead to catastrophic optical damage (COD), a well-known problem in semiconductor laser devices. However, we note that COD is usually observed at the laser facets [14] while, in the structure presented here, high powers are achieved only inside the grating so that a much higher COD threshold is to be expected. Indeed, similar waveguide structures have been irradiated with 800 W peak power levels with no reported degradation [15].

In conclusion we have reported the possibility of using light localization in a 1D PBG structures with a certain degree of randomness to enhance second-harmonic generation by many orders of magnitude. Preliminary measurements on mesa waveguides have been carried out and our simulations indicate that SHG enhancement due to localization should be observable in real systems. As a final comment regarding the possible applications of random perturbation induced localization, we note that nonlinear interactions based on waveguide photonic crystals require an extremely precise control on the periodicity and duty cycle of the crystal. On the basis of the technology available today we may expect a rather low yield from an industrial production. In fact, efficient second harmonic generation has still to be demonstrated in such waveguides. On the other hand, we show that use of randomly perturbed crystals will statistically produce localization and high SHG efficiency. If the sample is sufficiently long, localization will occur in each sample and the statistics regard only the SHG efficiency. Depending on the tolerance required on this value, we may expect the yield of a randomly perturbed PBG to be much higher than that from a perfectly ordered one.

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