Extreme events in resonant radiation from three-dimensional light bullets

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We report measurements that show extreme events in the statistics of resonant radiation emitted from spatiotemporal light bullets. We trace the origin of these extreme events back to instabilities leading to steep gradients in the temporal profile of the intense light bullet that occur during the initial collapse dynamics. Numerical simulations reproduce the extreme valued statistics of the resonant radiation which are found to be intrinsically linked to the simultaneous occurrence of both temporal and spatial self-focusing dynamics. Small fluctuations in both the input energy *and* in the spatial phase curvature explain the observed extreme behavior.

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The first direct measurement of an extreme or rogue wave in the ocean [1] has triggered a large number of studies and renewed interest in what used to be believed as a "sailor's tale." Extreme events have since been observed in many other systems illustrating the wide range of underlying dynamics that can lead to their appearance [2,3]. In an optical context, the recent observation that solitons with extreme spectral redshifts can emerge from an incoherent fiber supercontinuum [4] has attracted considerable attention and subsequent studies have highlighted the role of nonlinear noise amplification in the spontaneous emergence of highly localized structures [5,6] and the role of collisions in observing long tail statistics [7–9].

While most rogue wave studies in optics have focused on the $1+1$D fiber case (one spatial dimension and one temporal dimension), the emergence of extreme events during full three-dimensional (3D) propagation of light pulses with nonlinear spatiotemporal dynamics has been much less studied [10–13]. In the normal group velocity dispersion (GVD) regime, the combined action of spatial and temporal self-focusing leads to the formation of a dynamically stable filament characterized by an intense, localized peak surrounded by a weaker reservoir that continuously refuels the central core region [14]. In this regime, long tail statistics in the spectrum of a single filament similar to that of an incoherent fiber supercontinuum have been observed [10,11,13]. These rogue statistics have been associated to the formation of an X-shaped spatiotemporal intensity profile (as opposed to an extreme peak intensity value) [11] and no evidence of a coupling mechanism (such as that reported here) between *input* temporal and spatial fluctuations in the beam profile was observed to explain the extreme spectral intensity variations. More recently, extreme events in the multifilament regime where sporadic spatially localized structures emerge from mergers between filament strings have been measured experimentally in a gas cell [12]. In this regime the emergence of rogue waves appeared to be unrelated to any noise amplification mechanism but was attributed to atmospheric turbulence during the multiple filamentation stage.

In anomalous GVD, all three dimensions contribute to a spatiotemporal collapse with the formation of light bullets [15]. Although such light bullets lend themselves to a tempting analogy with 1D fundamental solitons, recent work has shown that in fact 3D light bullets correspond to a form of polychromatic (weakly localized) Bessel beam that emerges spontaneously during the collapse phase of an initially Gaussian-shaped wave packet [16]. Despite this difference, light bullets do exhibit remarkable similarities with 1D solitons: (i) they appear to propagate quasiundistorted without pulse splitting as observed in the normal GVD regime, (ii) temporal compression may occur in a fashion similar to soliton compression, and (iii) their propagation in the presence of higher-order dispersion perturbation is accompanied by the emission of a resonant radiation (RR) peak often referred to as a dispersive wave [15,17,18].

In this paper, we report on the observation of optical rogue waves associated with the emission of extreme amplitude RR during the formation of 3D light bullets in a nonlinear crystal. These rogue events are shown to originate from the coupling between spatial and temporal fluctuations of the input beam. In contrast to previous studies in the normal GVD regime, our experiments are performed in the anomalous GVD. Spectrally isolated RR is emitted with energies directly linked to the shock steepness of the light bullet that forms on the trailing edge of the pulse during the initial collapse phase. The initial noise present in the temporal and spatial beam profile (input energy and spatial phase curvature) is coupled and amplified during the dynamical evolution of the light bullet. Such coupling leads to dramatic changes in the shock steepness gradient and in turn in the energy of the emitted RR. Our experimental results are confirmed by full 3D numerical simulations, emphasizing the importance of both the initial temporal and spatial fluctuations and they highlight the central role of spatiotemporal coupling that underpins the manifestation of extreme events in a $3+1$D nonlinear optical system. We anticipate that similar mechanisms and extreme

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events can occur in other systems where temporal and spatial dynamics are coupled through a nonlinear collapse event.

The emission of resonant radiation was originally described in the $1+1$D context as a form of soliton instability that occurs in the presence of higher-order dispersion [18] and more recently interpreted in terms of cascaded four-wave mixing dynamics [8]. The frequency ω of the RR is determined by a dispersion relation $k(\omega) = k(\omega_0) + (\omega - \omega_0)/v$, where k represent the wave vector, and ω_0 and v are the central frequency and group velocity of the soliton, respectively [18]. The spectral signature of the RR is typically observed as a blueshifted emission in the normal GVD range. Similar RR emission predicted by the very same phase-matching relation occurs in 3D both for pulses in the anomalous and normal GVD regimes [19–21]. For normal GVD pumping, however, significant continuous spectral broadening may occur simultaneously and it is only for a pump in the anomalous GVD that the RR appears as an isolated spectral peak with strong similarities to the 1D soliton perturbation as observed in a variety of bulk media [17,22,23].

Light bullets resulting from the spatiotemporal collapse of an input Gaussian shaped pulse were generated following previous studies [16] by focusing 100 fs tunable pulses from an optical parametric amplifier (TOPAS-C, Light Conversion Ltd.) operating at 1 kHz repetition rate and for an input energy of 2.5 μJ into an 8-mm-long sapphire crystal with a $f = +100$ mm lens. The input energy was monitored with a pyroelectric detector. The energy and spectrum of the RR at the crystal output, on the other hand, were measured with a calibrated silicon photodiode and a fiber spectrometer (QE65000, Ocean Optics), respectively. The detection system was synchronized with the laser pulses allowing for single-shot characterization.

The light bullets produce a relatively intense and isolated resonant radiation in the visible region of the spectrum (see Fig. 1), in good agreement with the expected phase-matching relation as described above (see also Ref. [17]) and which shows a clear flickering and unstable behavior with occasional events of very high intensity. Selected single-shot RR spectra measured for an input wavelength of 1.9 μm [shown in Fig. 2(a)] illustrate how the RR emission wavelength is relatively stable but that there are indeed significant intensity fluctuations. We emphasize that the optical parametric amplifier operates in a relatively stable regime with a 1.02% standard deviation of the energy fluctuations. Full statistical analysis of the RR energy performed by integrating the individual RR spectra of 3000 laser shots reveals a highly skewed distribution as illustrated in Fig. 2(b) (the inset shows the full data in log scale). Defining the significant energy Ws as the mean of the highest one-third energies recorded [24,25] (directly adapted from the significant wave height criterion traditionally used in hydrodynamics [26]), events whose energy exceeds by a factor of 2 W_s can be considered as rogue [27]. In our measurements, we observe extreme fluctuations that exceed the significant energy W_s by more than a factor 5, a clear indication of the presence of rogue events. At first sight such extreme statistics in the RR energy may seem surprising as the propagation dynamics of the RR wave are typically linear due to the strong dispersion at the RR wavelength which results in the rapid walk-off from the main pump pulse and thus background-free propagation.

In order to clarify the origin of the extreme fluctuations observed in the RR energy, we plot in Fig. 3 the change in the RR energy as a function of the input pulse energy. The gray horizontal line marks the $2W_s$ limit and there are clearly many rogue events that lie above this line. The fact that the number of rogue events increases with the input energy fluctuations suggests that the RR energy is related to the input energy via a steep nonlinear transfer function [28] that converts the narrow (Gaussian) distribution of the input pulse energy into a distinct long-tailed distribution of the RR energy. To further confirm this hypothesis, we plot in Fig. 4 the histogram of the measured RR energy for decreasing input energy fluctuations (using data postselection) and we observe a gradual reduction in the number of events that populate the tail of statistical distribution and suggest deterministic dynamics for the occurrence of the rogue RR events. Significantly, for a fixed input energy, we

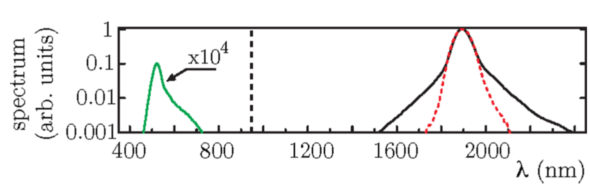

FIG. 1. (Color online) Experimentally measured spectrum (logarithmic scale) for a 1.9 μm wavelength pump pulse (dashed line is input spectrum) over the whole spectral range showing also the RR peak.

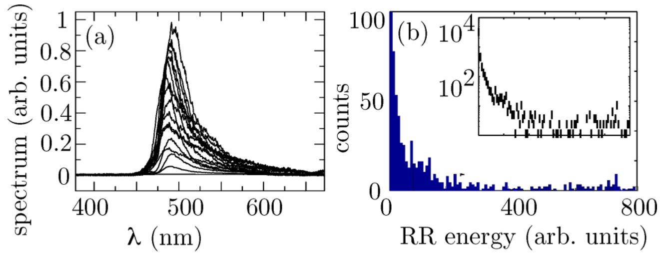

FIG. 2. (Color online) (a) Selection of 20 representative spectra recorded at nominally constant input energy and focusing conditions. (b) Statistical distribution of the RR energy (calculated from the integrated spectrum) and zoomed in along the vertical axis on the lower count rate range (0–100)—the full vertical range of data is shown in log scale in the inset.

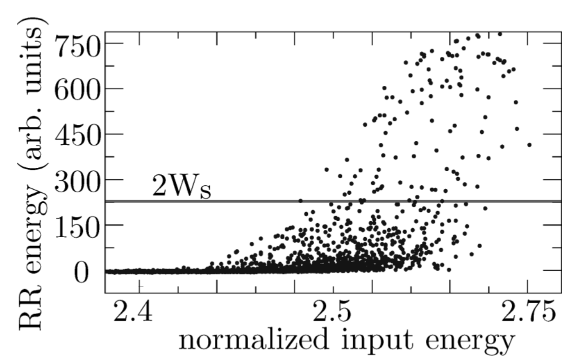

FIG. 3. Experimentally recorded RR energy as a function of input energy. The gray line indicates $2\times$ the significant energy mark W_s.

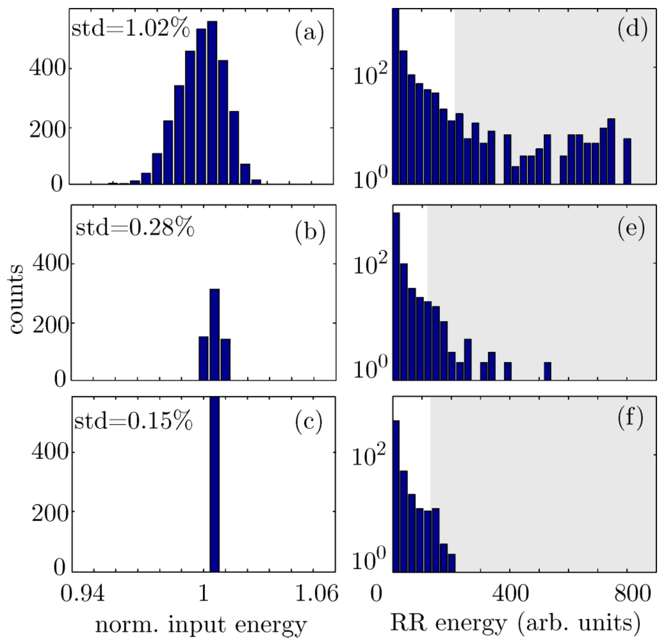

FIG. 4. (Color online) (a) Experimentally measured energy distribution of the input pump pulse (normalized to its mean value). (b), (c) The same data, postselected so as to keep only data with a smaller fluctuation (standard deviations of 0.28% and 0.15%, respectively) around a fixed mean value. (d)–(f) Histograms of the RR energy counts for the input energy distributions shown in (a)–(c): selecting input pump pulses with a narrow energy distribution clearly also narrows the RR energy L-shaped distribution. The shaded areas indicate events that are larger than $2W_s$.

still obtain a skewed (L-shaped) distribution of the RR energy, as can be seen from the dots taken along a single vertical line in the RR energy scatter of Fig. 3. This means that energy fluctuations alone do not explain our observations and that there must be at least one other relevant parameter that plays a role. These features are confirmed over the wider 1.7–2.2 μm wavelength range for pump pulses where RR with similar energy characteristics to those shown in Figs. 2 and 3 were observed.

The amplitude of the RR emitted by 3D light bullets is directly proportional to the steepness of the shock front that forms on the trailing edge of the collapsing input pulse as it converges towards the formation of a stable light bullet [20,29]. It is then natural to link the extreme fluctuations in the RR energy to a similar rogue behavior in the temporal gradient of the light bullet. In order to confirm the physical mechanism of the rogue statistics and gain a deeper understanding of the underlying dynamics, we have performed a series of numerical simulations based on the unidirectional pulse propagation equation (UPPE) [30] using the same input parameters as in the experiments. We did include quantum fluctuations of the input beam amplitude by adding half a photon with random phase per frequency bin of the simulation grid, yet these were found to have no effect on the emergence of rogue statistics. This is perhaps not surprising because there does not appear to be any nonlinear mechanism such as noise-seeded modulation instability that would be sensitive to such fluctuations but instead the RR emission is a coherent process seeded by the

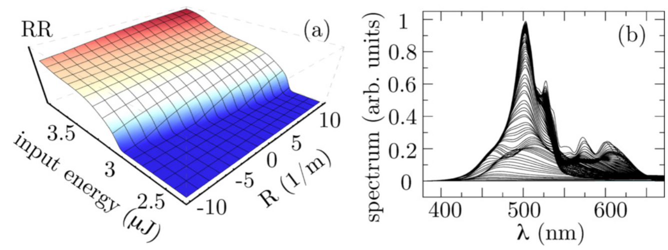

FIG. 5. (Color online) (a) Numerically simulated RR energy vs input energy and spatial phase curvature. (b) Superimposed RR spectra corresponding to all input energy and spatial phase curvature values used shown in (a).

pump spectral components themselves [19,20]. We therefore assume that the light bullet and associated RR emission is fully deterministic [28,31,32] so that the experimentally observed statistical behavior and rogue events may be reproduced by accounting for small fluctuations or changes in the input parameters which effectively leads to a stochastic system requiring statistical treatment. Most importantly, we allow for variations both in the input energy and in the profile of the spatial phase curvature. The extent of the spatial phase variation in the incident plane of the nonlinear crystal considered in the simulations is very weak, of the order of tens of meters at the laser output (i.e., before the actual experimental setup). Such small variations can be easily explained, e.g., from air turbulence along the beam path and from fluctuations from the OPA itself due to the nonlinear processes involved in the OPA operation (supercontinuum generation and optical parametric amplification).

The simulated RR energy is clearly a nonlinear function of the laser pulse energy and spatial phase curvature as shown in Fig. 5(a). Significantly, when we plot the RR spectra corresponding to each point of the 2D surface plot of Fig. 5(a) we find a striking resemblance between the simulated spectra and those observed experimentally [compare Fig. 5(b) with Fig. 2(a)] and see very large fluctuations in the RR energy. The histogram in Fig. 6(a) shows the statistics of the RR energy computed over the full parameter space of Fig. 5(a), assuming normal distributions of pulse energy and curvature fluctuations and is also in excellent agreement with the experimental results in Fig. 2(b). The projection of the RR energy on the plane of zero-phase curvature that gives the RR energy vs input

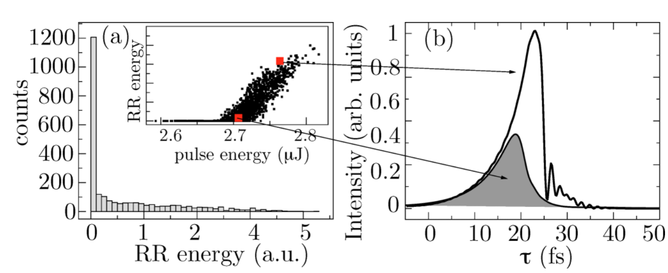

FIG. 6. (Color online) Numerical results: (a) statistics of the RR energy (inset shows RR energy versus input energy for zero-phase curvature). (b) On-axis temporal profiles of the light bullet corresponding to different steepnesses of the shock fronts, which give rise to RR with low energy and high energy.

pulse energy [see inset in Fig. 6(a)] shows reduced variations compared to the experimentally observed fluctuations of Fig. 3, which confirms the role of the initial spatial phase curvature fluctuations in observing rogue statistics.

The importance of the nonlinear coupling between the spatial and temporal dynamics in the propagation of light bullets leading to the observation of rogue statistics is highlighted in Fig. 6, where we show the temporal profile of the simulated light bullet at the crystal output in the case of a low and large value of the RR energy. The development of a steeper shock front on the trailing edge of the light bullet gives rise to a RR with a much higher energy, which occurs due to both variations in the energy and curvature. Yet, it is important to realize that it is *only* by including variations in the *energy* and the *spatial phase curvature* of the input beam that we are able to reproduce numerically the experimental results. We note that variations in the spatial phase curvature alone cannot explain the experimental behavior. This is because the steepness of the shock front is deeply rooted into the spatiotemporal coupling such that initial fluctuations in the input energy and spatial phase curvature lead to significantly larger discrepancies in the gradient of the shock steepness.

In conclusion, laser pulses propagating in a bulk nonlinear medium with anomalous GVD undergo a spontaneous reshaping into a 3D light bullet that, although being a distinctly nonsolitonic pulse [16], exhibits characteristic features, such as the emission of resonant radiation, that are commonly observed for 1D solitons. Yet the additional spatial degree of freedom leads to significantly richer dynamics caused by a spatiotemporal collapse associated with a steep temporal shock front formation on the trailing edge of the pulse. The formation of the shock front and in particular the corresponding gradient is extremely sensitive to small variations in the spatiotemporal profile of the input beam. Importantly, fluctuations in both the spatial and temporal input beam parameters are required to explain our observations. The space-time coupling of these small initial fluctuations is dramatically amplified by the shock formation, which results in rogue statistics for the amplitude of the resonant radiation. Evidence of such a coupling mechanism is an important step toward understanding and modeling of extreme events in multidimensional systems where the rogue dynamics arise from deterministic noise amplification through a multiparameter nonlinear transfer function. We believe that other classes of deterministic multidimensional systems where extreme value statistics are observed can be described in an analogous way and that the functional form and dimensionality of their corresponding transfer functions can be used to classify different types of rogue events. We anticipate that the interaction of multiple light bullets that could be spontaneously generated at higher input pulse powers would lead to similar collision processes to those suggested to be responsible for rogue waves in other nonlinear systems [8,9].

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